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LETTER TO THE EDITOR

A short note on the true self-avoiding walk

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Abstract. The scaling laws for the true self-avoiding walk and a Fokker–Planck equation for the end-to-end probability distribution are combined in order to calculate the scaling functions and the end-to-end distance in one dimension explicitly.

In a recent letter Obukhov (1984) obtained a partial differential equation for the dependence of the end-to-end probability distribution $P_N(x)$ on the number of steps N of the walk and the position x for the one-dimensional true self-avoiding walk (TSAW; Amit *et al* 1983) in the case of small self-avoidance parameter g:

$$\frac{\partial P_N(x)}{\partial N} = \frac{1}{2}\frac{\partial^2 P_N(x)}{\partial x^2} - \frac{\partial}{\partial x}(b(x)P_N(x)) \tag{1}$$

(in the limit of large N the variables N and x can be treated as continuous variables). The first term on the right-hand side of the Fokker–Planck equation (1) represents a random diffusion whereas the second term describes a drift, which for the TSAW is defined by the gradient of the total number of previous visits $n_N(x)$ of point x (the TSAW tries to avoid places already visited):

$$b(x) = -2g\,\partial n_N(x)/\partial x. \tag{2}$$

In this letter the consequences of these equations will be analysed in more detail.

Equations (1) and (2) can be combined with the scaling laws (Pietronero 1983, Bernasconi and Pietronero 1984)

$$P_N(x) = (1/R_N)f(x/R_N) \tag{3}$$

$$n_N(x) = (N/R_N)h(x/R_N) \tag{4}$$

where

$$R_N = \lambda_g N^\nu \tag{5}$$

is the root-mean-square displacement and f and h are normalised universal functions. Using the relation

$$n_N(x) = \int_0^N \mathrm{d}n\, P_N(x) \tag{6}$$

one can express the function f in terms of h:

$$f(z) = (1-\nu)h(z) - \nu z h'(z). \tag{7}$$

Inserting the scaling laws (3) and (4) into Obukhov's equations (1) and (2) one obtains (with $z=x/R_N$)

$$-\nu(df(z)+zf'(z)) = \frac{N^{1-2\nu}}{2\lambda_g^2}[f''(z)+(d-1)f'(z)/z] + 2g\frac{N^{2-\nu(d+2)}}{\lambda_g^{d+2}}(h''(z)f(z)+h'(z)f'(z)+(d-1)h'(z)f(z)/z) \tag{8}$$

where the preceding equations have been generalised to arbitrary dimension d (Pietronero 1983). Provided that the asymptotic behaviour of the TSAW is determined by the self-avoidance, the second term on the right-hand side of equation (8) has to be of the same order of magnitude as the left-hand side (in the limit $N\to\infty$) and Pietronero's (1983) famous result $\nu=2/(d+2)$ follows from (8) (this result was re-derived by Family and Daoud (1984)). However, for $\nu=\frac{1}{2}$ ($d=2$) the first term on the right-hand side of equation (8) becomes relevant and above the upper critical dimension $d_c=2$ one finds pure random-walk behaviour (the second term is negligible for $N\to\infty$). Below $d_c=2$ the diffusion term with $\nu=\frac{1}{2}$ can be neglected except for $g=0$ (in agreement with the renormalisation group picture (de Queiroz *et al* 1984)).

Only the case $d=1$ will be considered in the following. By integration of equation (8) in this case one obtains (the constant of integration vanishes because of the normalisation of f and h)

$$h'(z)f(z)=-(\lambda_g^3/3g)zf(z). \tag{9}$$

Because the universal functions f and h have to be independent of g, one concludes from (9) that

$$\lambda_g \sim g^{1/3} \tag{10}$$

(for arbitrary $d<d_c$ one concludes from (8) that $\lambda_g \sim g^{1/(2+d)}$). This relation was confirmed by Rammal *et al* (1984) by means of a Monte Carlo simulation.

Using the fact that due to equation (7) one has $f(z)=0$ for $h(z)=0$ (the more general solution $h(z)=c\sqrt{z}$ compatible with $f(z)=0$ is unphysical) one obtains the normalised, positive and continuous solution of (9)

$$h(z)=\begin{cases}\dfrac{3}{4z_0}\left(1-\dfrac{z^2}{z_0^2}\right) & \text{for } |z|<z_0\\ 0 & \text{for } |z|\geqslant z_0\end{cases} \tag{11}$$

where the constant z_0 is defined by

$$z_0=(\tfrac{9}{2}g)^{1/3}/\lambda_g. \tag{12}$$

If one calculates R_N by means of equations (3), (7) and (11) the free constants z_0 and λ_1 are fixed for reasons of consistency:

$$z_0=(\tfrac{15}{7})^{1/2}\approx 1.46 \qquad \lambda_1=(\tfrac{1029}{500})^{1/6}\approx 1.13. \tag{13}$$

This value for λ_1 is slightly but appreciably smaller than the Monte Carlo value $\lambda_1=1.50\pm0.02$ (estimated from a simulation with $g=0.1$). Furthermore, the form (11) of $h(z)$ disagrees with Monte Carlo data (see figure 3 of the paper by Bernasconi and

Pietronero (1984)) and produces a discontinuous function $f(z)$ which increases as z^2 around $z = 0$ and jumps to zero at $|z| = z_0$.

In conclusion, the Fokker-Planck equation derived by Obukhov (1984) predicts (combined with scaling laws) the value of the exponent ν in $R_N = \lambda_g N^\nu$ very precisely and the constant λ_g quite well, but fails to describe the correct form of the scaling functions in one dimension. This might be for two reasons. Firstly, it might be necessary to keep higher-order terms in g in the derivation of the Fokker-Planck equation (1). Secondly, instead of using the distribution $n_N(x)$ one should use a conditional distribution of two variables x and y in the derivation of (1) and (2), namely the number of previous visits of point x provided that the TSAW ends at point y after N steps.

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References

Amit D, Parisi G and Peliti L 1983 *Phys. Rev.* B **27** 1635
Bernasconi J and Pietronero L 1984 *Phys. Rev.* B **29** 5196
de Queiroz S L A, Stella A L and Stinchcombe R B 1984 *J. Phys. A: Math. Gen.* **17** L45
Family F and Daoud M 1984 *Phys. Rev.* B **29** 1506
Obukhov S P 1984 *J. Phys. A: Math. Gen.* **17** L7
Pietronero L 1983 *Phys. Rev.* B **27** 5887
Rammal R, Angles d'Auriac J C and Benott A 1984 *J. Phys. A: Math. Gen.* **17** L9